Practice §14.4: Tangent Planes

Instructions: Complete each of the following exercises for practice.

1. Find an equation of the tangent plane to the given surface at the given point.
   (a) $z = 2x^2 + y^2 - 5y; \quad P = (1, 2, -4)$
   (b) $z = (x+2)^2 - 2(y-1)^2 - 5; \quad P = (2, 3, 3)$
   (c) $z = e^{x-y}; \quad P = (2, 2, 1)$
   (d) $z = \dfrac{x}{y^2}; \quad P = (-4, 2, -1)$
   (e) $z = x\sin(x+y); \quad P = (-1, 1, 0)$
   (f) $z = \ln(x - 2y); \quad P = (3, 1, 0)$
2. Compute the linearization $L(x, y)$ of f at P.
   (a) $f(x, y) = 1 + x\ln(xy - 5); \quad P = (2, 3)$
   (b) $f(x, y) = \sqrt{xy}; \quad P = (1, 4)$
   (c) $f(x, y) = x^2 e^y; \quad P = (1, 0)$
   (d) $f(x, y) = \dfrac{1+y}{1+x}; \quad P = (1, 3)$
   (e) $f(x, y) = 4\arctan(xy); \quad P = (1, 1)$
   (f) $f(x, y) = y + \sin(\frac{x}{y}); \quad P = (0, 3)$
3. Compute the differential of the function.
   (a) $z(x, y) = e^{-2x}\cos(2\pi y)$
   (b) $u(x, y) = \sqrt{x^2 + 3y^2}$
   (c) $m(p, q) = p^5 q^3$
   (d) $T(u, v, w) = \dfrac{v}{1 + uvw}$
   (e) $R(\alpha, \beta, \gamma) = \alpha\beta^2\cos(\gamma)$
   (f) $L(x, y, z) = xze^{-y^2 - z^2}$
4. Suppose function $f(x, y)$ is differentiable at (a, b). Prove f is continuous at (a, b).